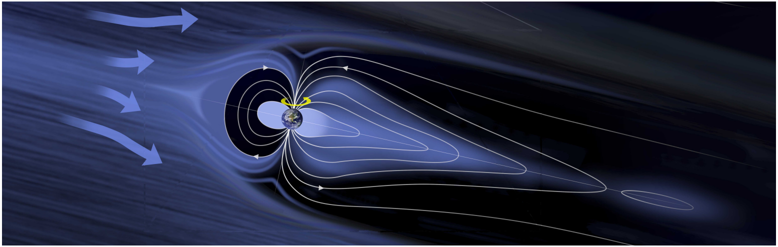
Space Weather
Lecture 7: Earth's Magnetic Field



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Earth's undisturbed magnetic field

- Magnetic potential

$$\Phi(\mathbf{r}) = \frac{\mu_0}{4\pi r^3}\mathbf{M} \cdot \mathbf{r}$$

- The Earth's best fit dipole moment is about $M = 8 \times 10^{22}$ A · m^2, r is the radius and μ_0 is the magnetic permeability.

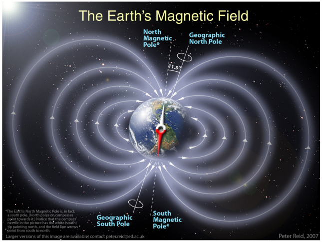
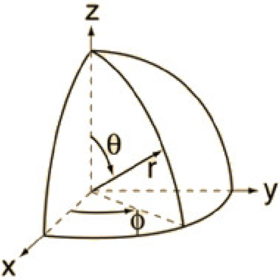


image from www.scifun.ed.ac.uk

- Magnetic field is the derivative of the potential

$$\mathbf{B}(\mathbf{r}) = -\nabla\Phi(\mathbf{r})$$

Spherical polar coordinates and magnetic potential



r... radius
θ... colatitude, 0 to π, (degrees from north pole)
ϕ... longitude, 0 to 2π

- Gradient operator

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r}\frac{\partial f}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\right)$$

- Magnetic field in spherical polar coordinates

$$\mathbf{B}(\mathbf{r}) = (B_r, B_\theta, B_\phi)$$

- If the Earth's dipole moment is aligned along the z-axis,

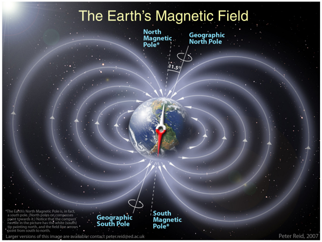
$$\Phi(\mathbf{r}) = \frac{\mu_0}{4\pi r^3}\mathbf{M}\cdot\mathbf{r} = \frac{\mu_0 M r\cos\theta}{4\pi r^3} = \frac{\mu_0 M\cos\theta}{4\pi r^2}$$

one can calculate the magnetic field at any point...

Undisturbed Earth's magnetic field

- Three componets

$$
\begin{aligned}
B_r(r,\theta,\phi) &= -\frac{2\mu_0 M \cos\theta}{4\pi r^3} \\
B_\theta(r,\theta,\phi) &= -\frac{\mu_0 M \sin\theta}{4\pi r^3} \\
B_\phi(r,\theta,\phi) &= 0
\end{aligned}
\qquad (1)
$$



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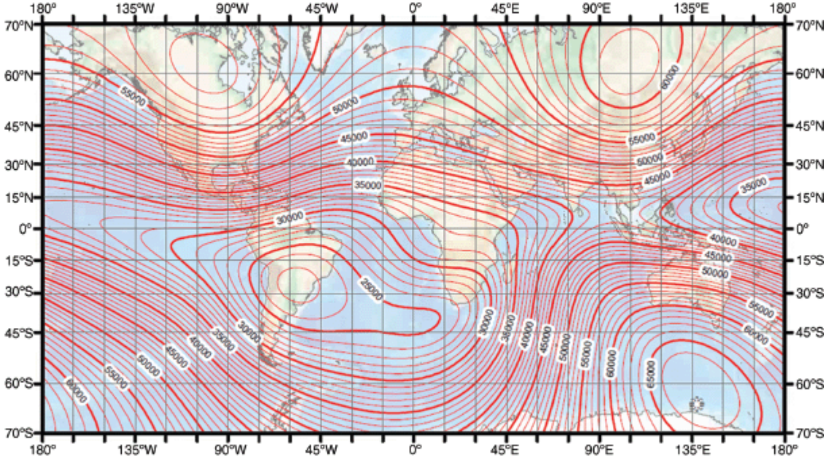
- Total field

$$B(r,\theta,\phi) = \sqrt{B_r^2 + B_\theta^2 + B_\phi^2} = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

- At the pole, $B_r(r,0^\circ,\phi) = -\frac{\mu_0 M}{2\pi r^3},\ B_\theta(r,0^\circ,\phi) = 0$
- At the equator, $B_r(r,90^\circ,\phi) = 0,\ B_\theta(r,90^\circ,\phi) = -\frac{\mu_0 M}{4\pi r^3}$
- Magnitude of the total field at the pole is twice as strong as at the equator.

Magnetic field intensity

- Total magnetic field intensity

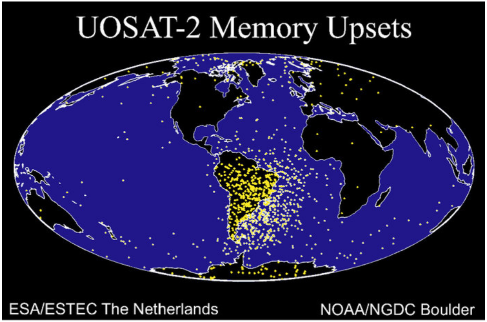


From Maus et al., 2010

South Atlantic Anomaly

- This chart maps the location of memory (static random-access memory based on semiconductor) failures, in yellow, for satellite UoSAT-2.
- They happened much more frequently as it passed through the South Atlantic Anomaly.
- During solar storms, objects passing through the anomaly are much more strongly affected by damaging cosmic rays.

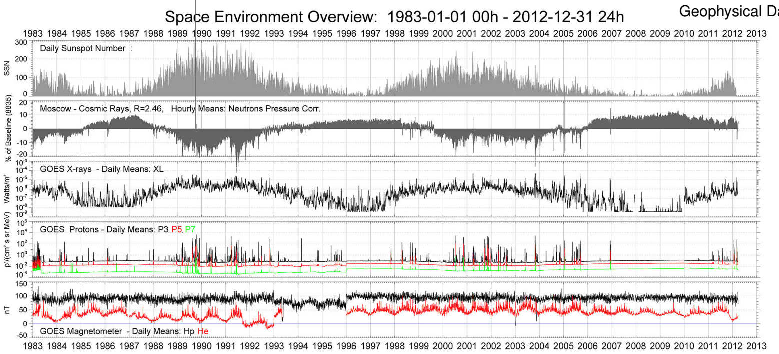
Credit: M. A. Shea, Geophysics Directorate, Philips Laboratory



Cosmic Rays

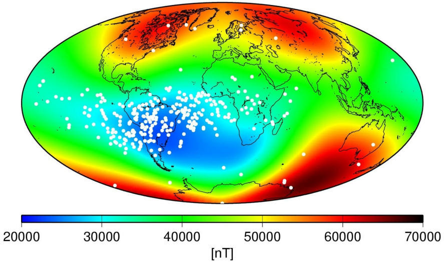
- Cosmic rays are high-energy ($\sim$speed of light) particles (mainly p)
- They originate from the Sun and from outside of our galaxy (galactic cosmic rays)
- Cosmic rays can alter circuit components in electronic devices, causing transient errors (such as corrupted data in memory devices or incorrect performance of CPUs)

Credit: ”Extreme Space Weather Events”, National Geophysical Data Center.

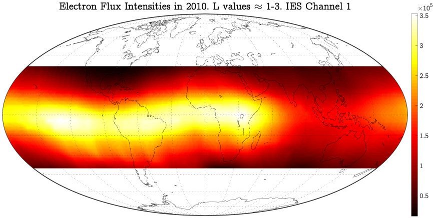


South Atlantic Anomaly

- Electron intensities derived from the Cluster/RAPID observations.



Credit: ESA



Credit: Smirnov&Kronberg

Dipole line equation of the magnetic field

Let us consider a segment $\mathrm{d}\mathbf{s}$ along the magnetic field line, where

$$d\mathbf{s} = \hat{r}\mathrm{d}r + \hat{\theta}r\mathrm{d}\theta + \hat{\phi}r\sin\theta\mathrm{d}\phi.$$

Let

$$\mathbf{B} = \hat{r}B_r + \hat{\theta}B_\theta + \hat{\phi}B_\phi$$

Since $\mathbf{B} \parallel \mathrm{d}\mathbf{s}$, it yields

$$\frac{\mathrm{d}r}{B_r} = \frac{r\mathrm{d}\theta}{B_\theta} = \frac{r\sin\theta\mathrm{d}\phi}{B_\phi} = \frac{\mathrm{d}s}{B}$$

Using Eq. (1) we get

$$\frac{\mathrm{d}r}{2\cos\theta} = \frac{r\mathrm{d}\theta}{\sin\theta} \quad \text{and then} \quad \frac{\mathrm{d}r}{r} = 2\frac{\mathrm{d}\sin\theta}{\sin\theta}$$

Solving this equation and assuming that $r(\theta = \pi/2) = r_{\mathrm{eq}} = LR_{\mathrm{E}}$, we get $r(\theta) = r_{\mathrm{eq}}\sin^2\theta$ – equation of dipole magnetic field line

Dipole line equation of the magnetic field

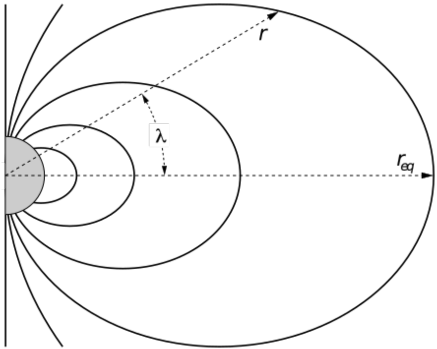
Dipole line equation:

$$r = r_{\text{eq}} \cos^2 \lambda$$

where λ is magnetic latitude, r_{eq} is the distance to the equatorial crossing of the field line. L-shell parameter $L = r_{\text{eq}}/R_{\text{E}}$:

$$\cos^2 \lambda_{\text{E}} = L^{-1},$$

the latitude, λ_{E}, where a field line with a given $L-$value intersects the Earth's surface.



Particle motion: adiabatic invariants

- Gyro motion:

$$\mu = mv_{\perp}^2 / B$$

$\sim 10^{-3}$ sec

- Bouncing:

$$J_1 = \int mv_{\|} \mathrm{d}s$$

$\sim 10^{0}$ sec

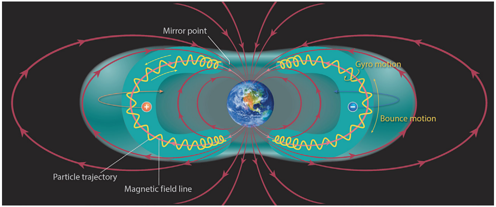
- Drift:

$$\Phi = \int B \mathrm{d}A,$$

where $\mathrm{d}A$ is a surface element

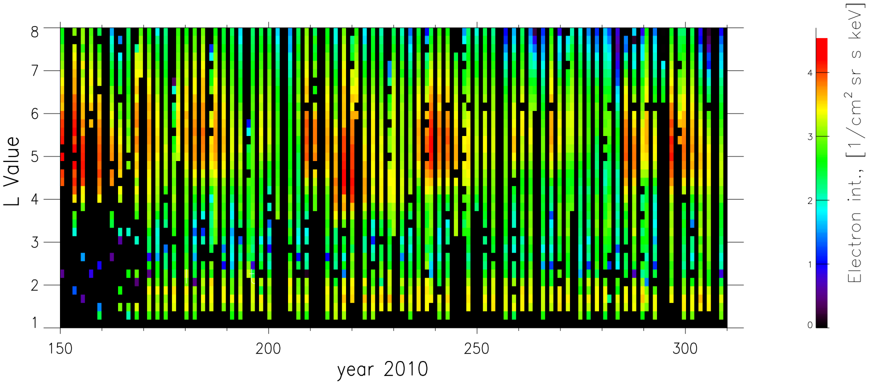
$\sim 10^{3}$ sec

Ilie 2020



Baumjohann&Treumann

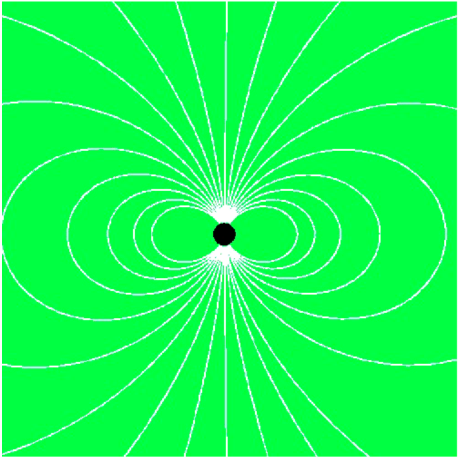
Application of L-shell: radiation belts



Kronberg+16

Magnetosphere

- Solar wind hits dipolar field of the Earth with supersonic speed



Magnetopause shape

- At stand-off distance, R_{mp}, solar wind dynamic ram pressure is equal to the pressure of the geomagnetic field (here dipolar):

$$n_{sw} m_i v_{sw}^2 = \frac{K B_{\mathrm{E}}^2}{2\mu_0 R_{mp}^6}$$

where n_{sw} ... is the solar wind density, v_{sw} ... is the solar wind speed, K ... constant accounting for deviation from dipolar magnetic field, B_{E} ... the magnetic field of the Earth, m_{i} ... mass of ion

- At flanks, the thermal pressure is equal to the pressure of the geomagnetic field (here dipolar):

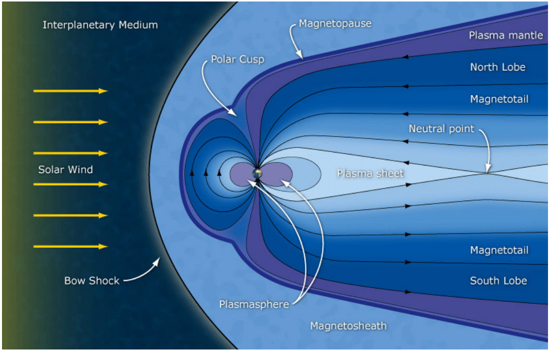
$$\gamma n_{sw} k_B T_{sw} = \frac{K B_{\mathrm{E}}^2}{2\mu_0 R_{mp}^6}$$

where γ ... is the ratio of specific heat or the polytropic index, k_B ... the Boltzmann constant, T_{sw} ... the solar wind temperature.

Position of the magnetopause

- Position of the magnetopause at nose ($\simeq 10 R_\mathrm{E}$ during quiet time)
- Under very active solar wind conditions may move inside geostationary orbit

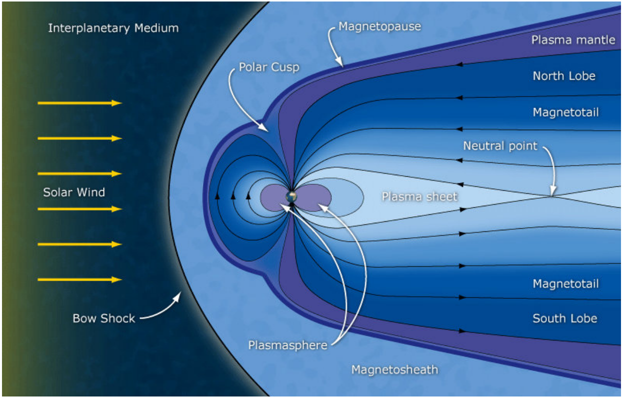
$$R_{mp} = \left(\frac{K B_\mathrm{E}^2}{2\mu_0 n_{sw} m_i v_{sw}^2}\right)^{1/6} [R_\mathrm{E}]$$



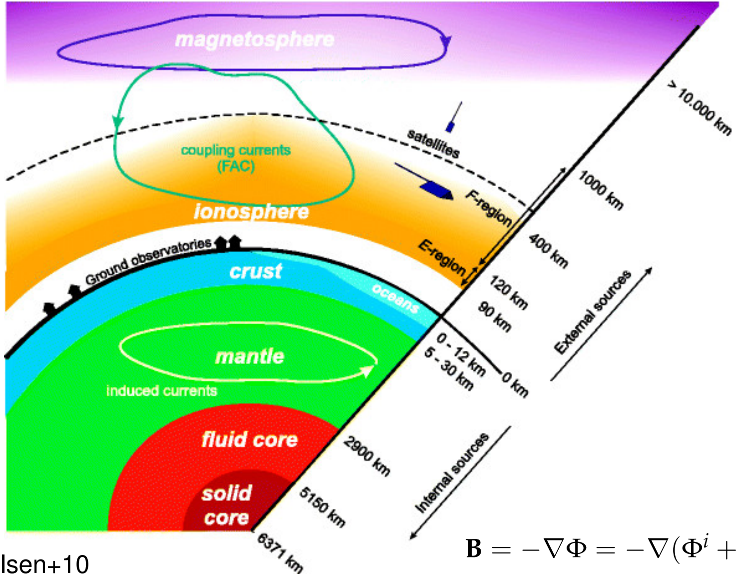
Magnetosphere

- Position of the magnetopause at flanks ($\simeq 14 R_{\mathrm{E}}$ during quiet time)

$$R_{mpf} = \left(\frac{K B_{\mathrm{E}}^2}{2\mu_0 \gamma n_{sw} k_B T_{sw}}\right)^{1/6} [R_{\mathrm{E}}]$$



Sources of the magnetic field



Olsen+10

$$\mathbf{B} = -\nabla\Phi = -\nabla(\Phi^i + \Phi^e)$$

Generalized Planetary Magnetic Fields (short scale dynamics is not included)

$$\Phi^i(r,\theta,\phi) = a\sum_{m=0}^{\infty}\sum_{n=1}^{n}[r/a]^{-n-1}P_n^m(\cos\theta)[g_n^m\cos(m\phi)+h_n^m\sin(m\phi)]$$

$$\Phi^e(r,\theta,\phi) = a\sum_{m=0}^{\infty}\sum_{n=1}^{n}[r/a]^{n}P_n^m(\cos\theta)[G_n^m\cos(m\phi)+H_n^m\sin(m\phi)],$$

where a is the planet's radius.

- $P_n^m(\cos\theta)$ are Legendre functions with Schmidt normalization:

$$P_n^m(\cos\theta) = N_{nm}(1-\cos^2\theta)^{m/2}d^mP_n(\cos\theta)/d(\cos\theta)^m,$$

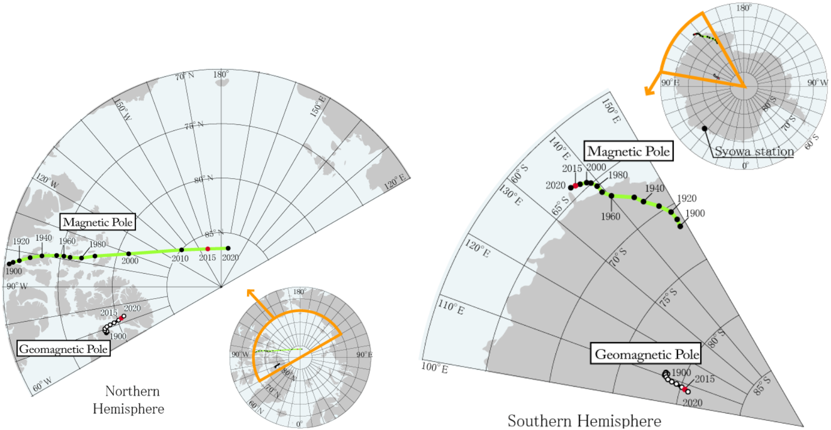
  where $P_n(\cos\theta)$ is the Legendre function, and $N_{nm}=1$ when $m=0$, and $[2(n-m)!/(n+m)!]^{1/2}$ otherwise.
- Dipole approximation is when $n=1, m=0,1$:

$$M = a^3[(g_1^0)^2+(g_1^1)^2+(h_1^1)^2]^{1/2}$$

- and the tilt of the dipole moment to the rotation axis is $\alpha = \cos^{-1}(g_1^0/M)$
- These coefficients are functions of time.
- International Geomagnetic Reference Field (IGRF) is based on this approach (updated every 5 years, 14th edition is released in 2024/12).

Magnetic field change

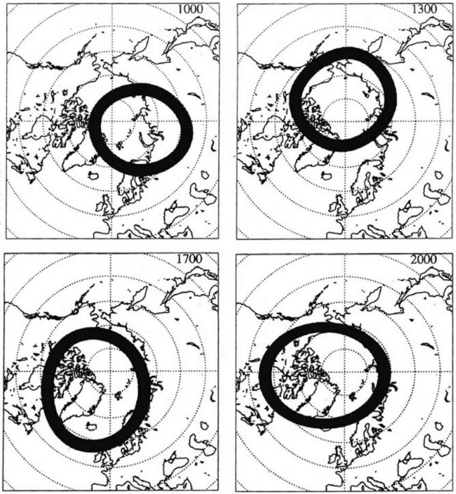
- Maps of the Arctic and Antarctic showing the area where the magnetic and geomagnetic poles have been situated during the last century.



From `http://wdc.kugi.kyoto-u.ac.jp/poles/polesexp.html`

Magnetic field change

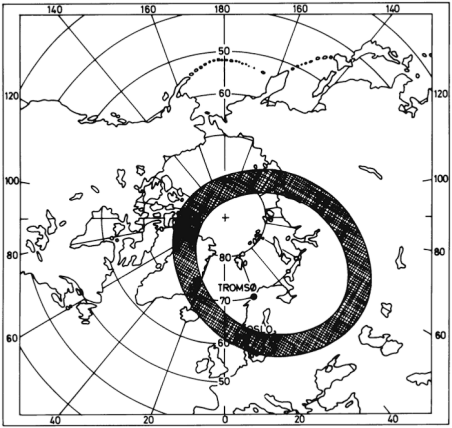
- The position of the auroral oval for four periods in historic time



From: Brekke

Magnetic field change

- Prediction of the position of the auroral oval in AD 2300

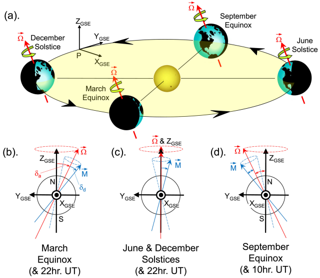


After: Oguti, 1994

Coordinate Systems

- The Geocentric Solar Ecliptic (GSE) system has its x-axis pointing from the Earth toward the Sun, its y-axis is in the ecliptic plane pointing toward dusk. Its z-axis is parallel to the ecliptic pole.

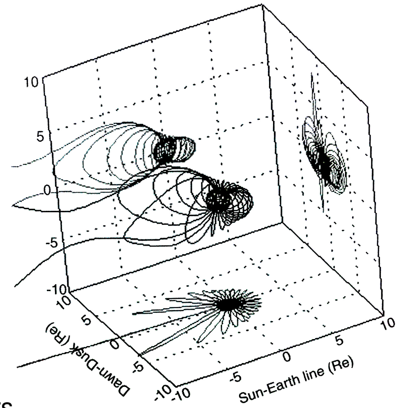
  Use: display satellite trajectories, IMF and solar wind observations.
- The Geocentric Solar Magnetospheric (GSM) system has its x-axis as GSE. The y-axis is $\perp$ to the Earth's magnetic dipole, so that the $x - z$ plane contains the dipole axis. The positive z-axis is in the same sense as the northern magnetic pole. The difference with GSE is simply rotation about the x-axis.
  Use: displaying magnetopause and shock boundary positions, magnetotail magnetic fields. It reduces 3D motion of the dipole.



Lockwood et al. 2016

External magnetic field model (short scale dynamics is included!)

- Geomagnetic field line traces computed using the 2004 Tsyganenko model for 57° geographic latitude and quiet magnetic conditions: dynamic pressure D_p=3 nPa, IMF B_y=0 nT, B_z=0 nT and magnetic disturbance index Dst= 5 nT.

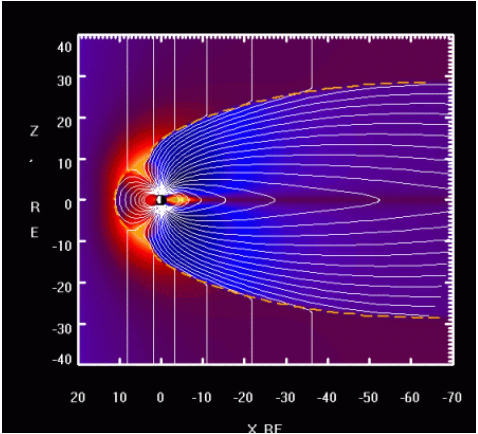


Credit: Menk&Waters

Diurnal and yearly wobbling of the geodipole

- The background color coding displays the distribution of the scalar difference ΔB between the total model magnetic field and that of the Earth's dipole alone. Yellow and red colors correspond to the negative values of ΔB. Black and blue colors indicate a compressed field.

The Earth rotation axis tilts 23.5° from Z axis in GSE causes seasonal variation. The dipole axis inclines $\sim$11.5° from the rotation axis and leads to the diurnal variation.



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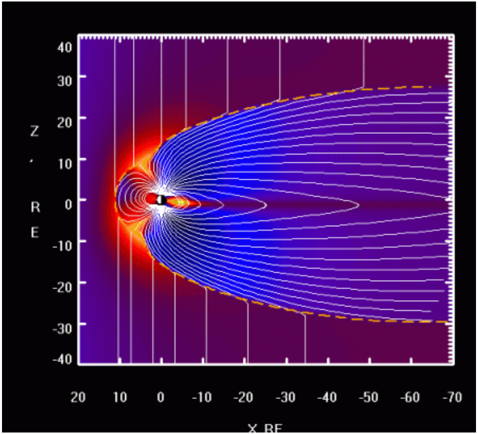

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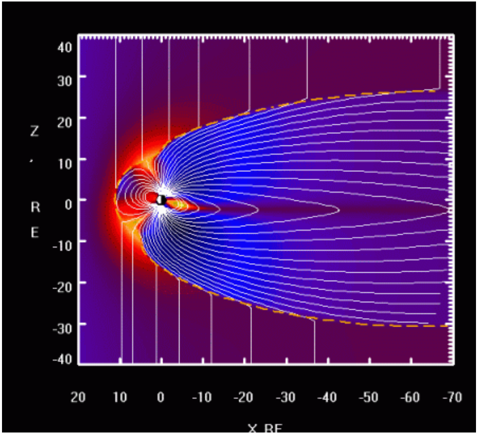

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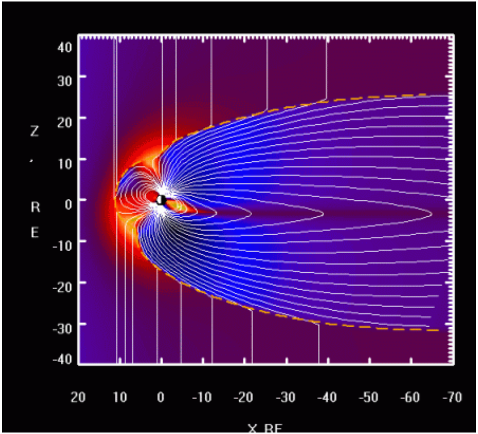

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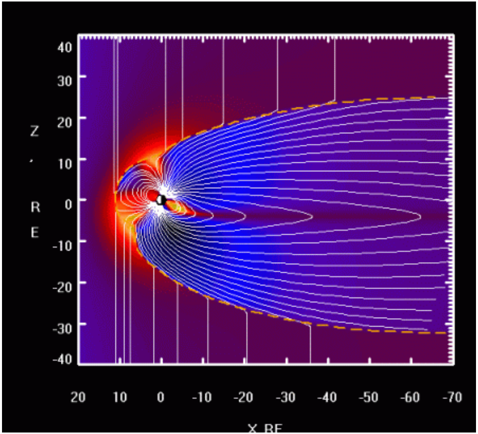

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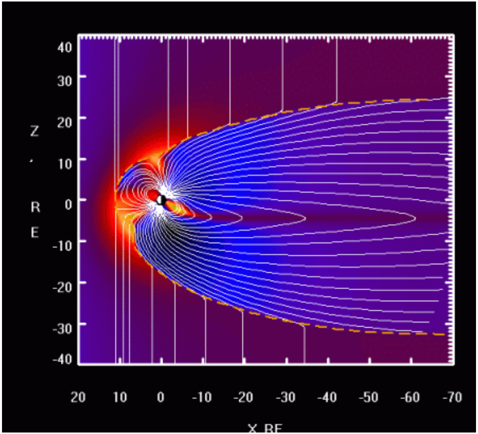

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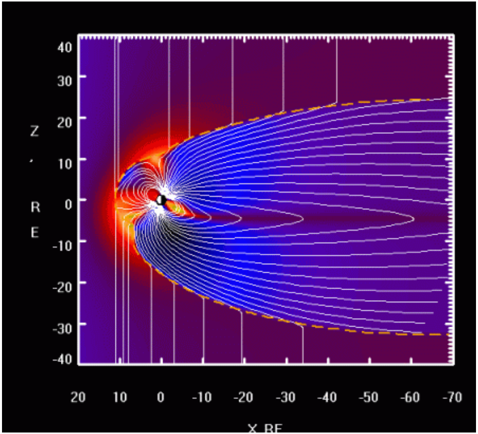

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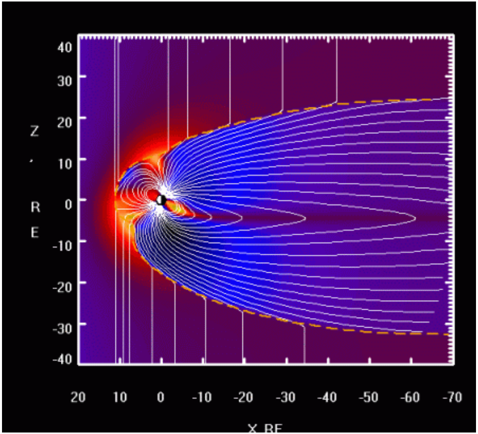

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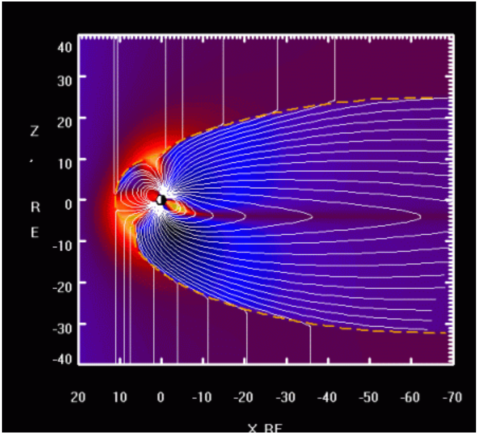

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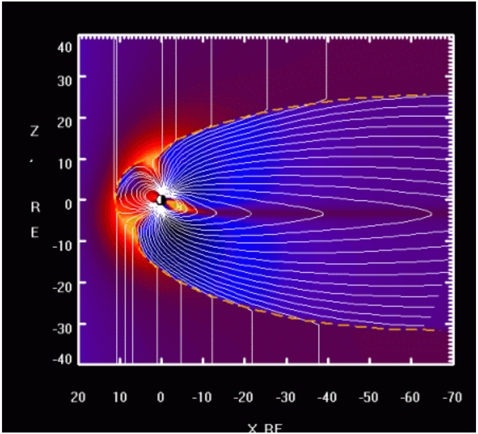

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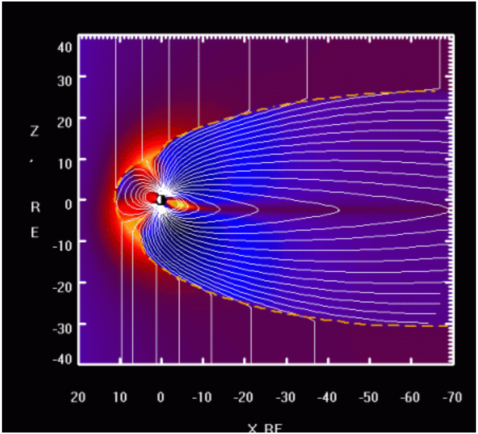

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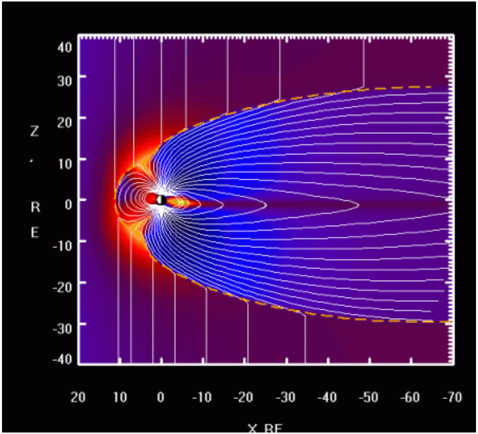

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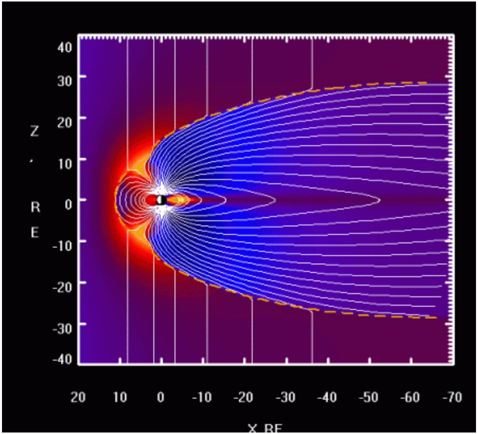

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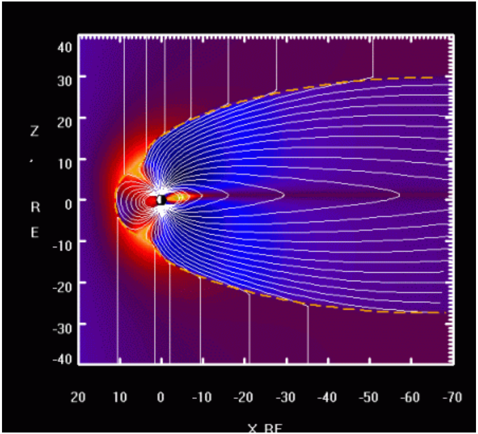

Diurnal and yearly wobbling of the geodipole

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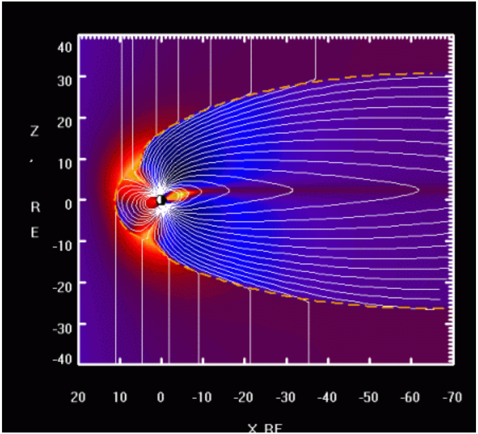

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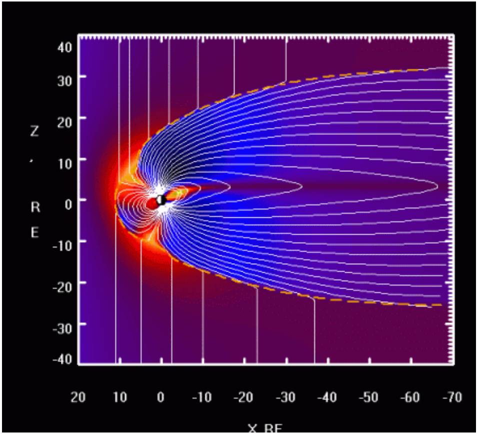

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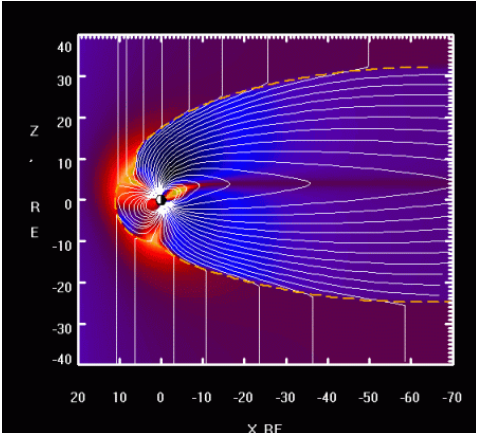

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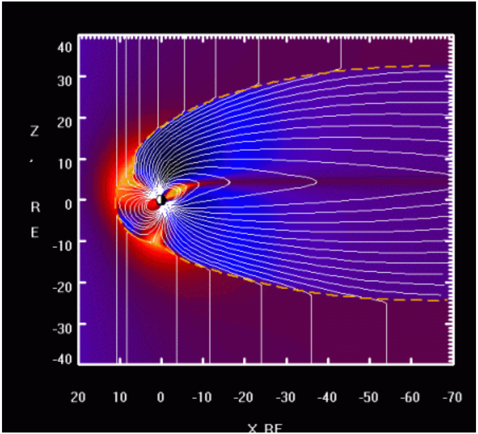

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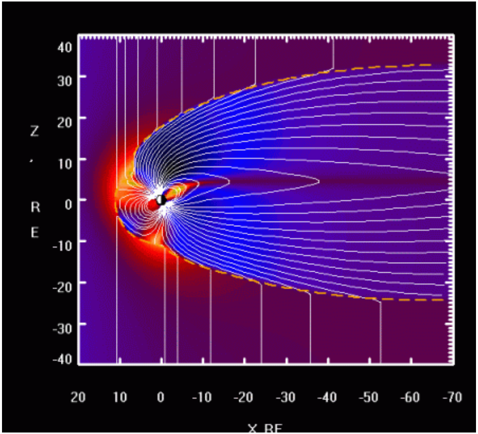

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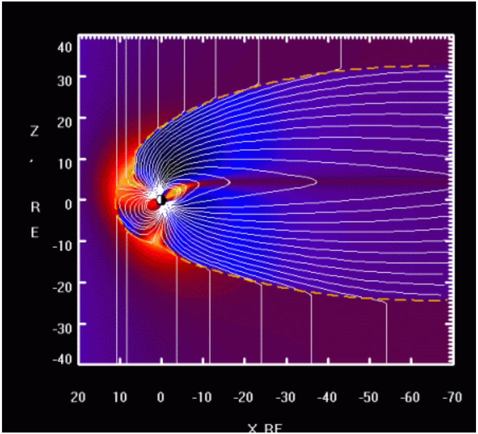

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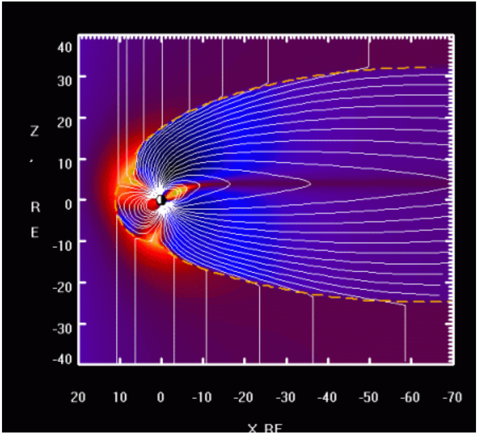

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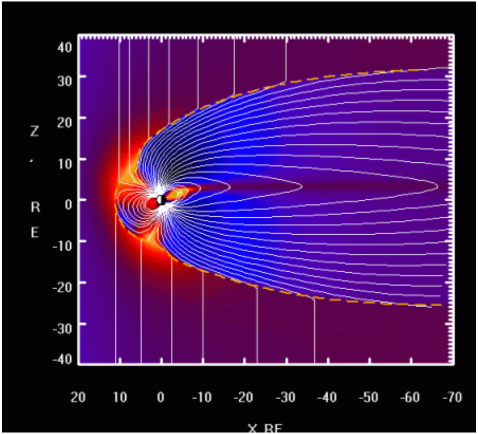

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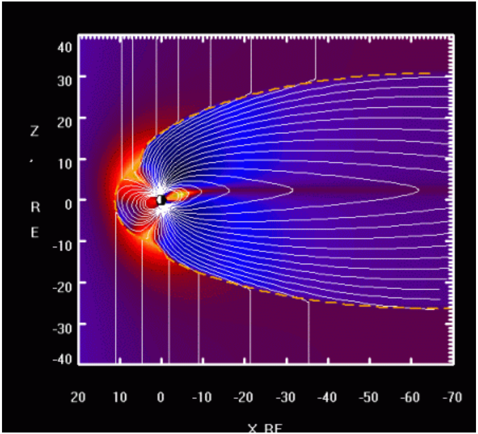

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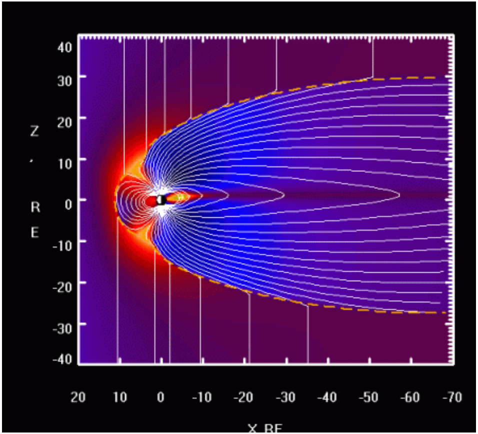

Summary

- In the inner magnetosphere (radiation belts) the magnetic field can be approximated by a dipole model.
- External magnetic field has complicated structure which depends on the solar wind dynamics.
- One has to carefully choose the coordinate system while working with the magnetic field of the Sun or the magnetosphere.

Literature

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